



Christ Church
Grammar School

2017
UNIT TEST 5

MATHEMATICS SPECIALIST Year 12

Section One: Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

Question 1**(5 marks)**

Determine the equation of the tangent to the curve defined $x^2 - xy + y^3 = 5$ at the point $(2, -1)$.

Question 2**(6 marks)**

A salad, which is initially at a temperature of 25°C , is placed in a refrigerator that has a constant temperature of 3°C . The cooling rate of the salad is proportional to the difference between the temperature of the refrigerator and the temperature, T , of the salad. That is, T satisfies the equation

$$\frac{dT}{dt} = -k(T - 3)$$

(a) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. (2 marks)

(b) The temperature of the salad is 11°C after 10 minutes.
Find the temperature of the salad after 15 minutes. (4 marks)

Question 3**(4 marks)**

A particle is moving in simple harmonic motion in a straight line. Its maximum speed is 3 m/s and its maximum acceleration is 6 m/s^2 .

Find the amplitude and the period of the motion.

End of questions



MATHEMATICS SPECIALIST Year 12

Section Two:

Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes

Working time for this section: 30 minutes

Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
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4. It is recommended that **you do not use pencil**, except in diagrams.

Question 4**(6 marks)**

Consider the differential equation

$$\frac{dy}{dx} = \frac{-x}{2y}, y > 0$$

with initial values $x = 2$ and $y = 3$.

- (a) Use Euler's method with a step size of 0.2 in the values of x to determine an approximate value of y when $x = 2.6$. (3 marks)

- (b) Determine the exact value of y when $x = 2.6$. (3 marks)

Question 5

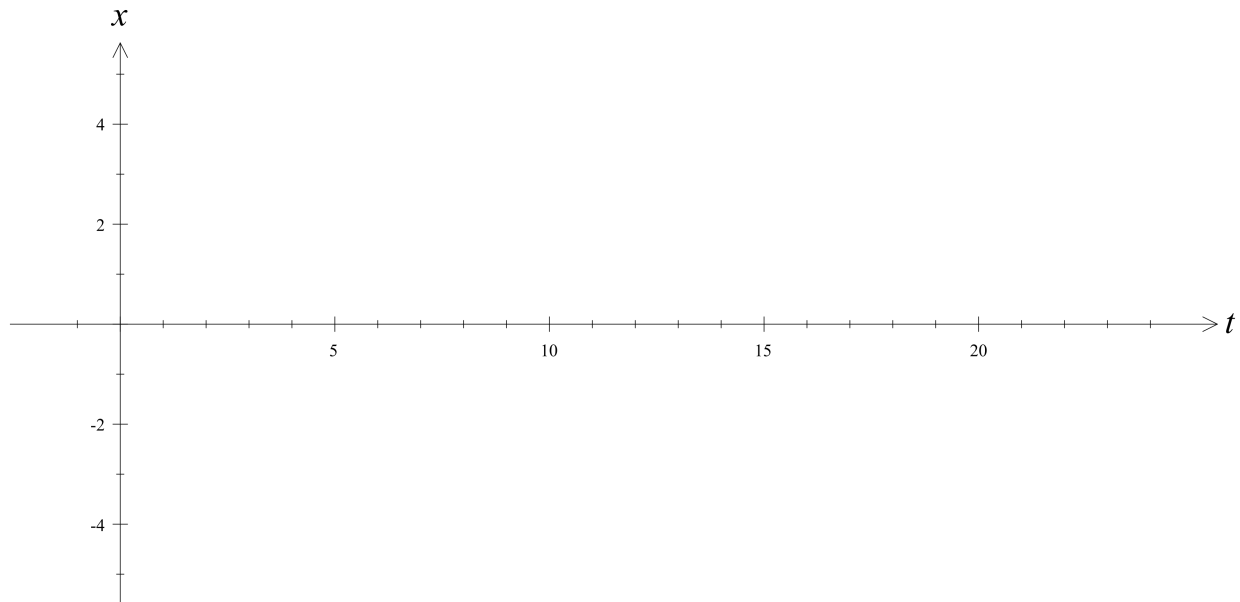
(9 marks)

The depth of water in a harbour, above and below the mean depth, is an example of simple harmonic motion.

In a particular harbour the low tide depth of 9 metres is recorded at 8 am one morning and the next high tide is expected to record a depth of 17 metres at 3.30 pm later that same day.

A particular container ship requires a depth of at least 11 metres for safe entry into the harbour, for unloading at the dock side and for leaving.

- (a) For this particular day, sketch the graph of the depth of water, x (metres), above and below the mean depth, as a function of the time t (hours) in the coordinate system below. (4 marks)



- (b) On this particular day, what proportion of the day will it be safe for the container ship to engage in these activities? (5 marks)

Question 6**(10 marks)**

A TV channel has estimated that if it spends \$ x on advertising a particular program it will attract a proportion $y(x)$ of the potential audience for the program, where

$$\frac{dy}{dx} = ay(1 - y)$$

for some constant $a > 0$.

- (a) Show that $\int \frac{dy}{y(1-y)} = \ln\left(\frac{y}{1-y}\right) + c$ for some constant c , where $0 < y < 1$.
(3 marks)

- (b) Hence, or otherwise, show that $y(x) = \frac{1}{ke^{-ax} + 1}$ for some constant $k > 0$.
(3 marks)

The TV channel knows that if it spends no money on advertising the program then the audience will be one-tenth of the potential audience. It also knows that if it spends \$100000 on advertising the program then the audience will be half of the potential audience.

- (c) Determine the values of a and k . (4 marks)

Question 7**(5 marks)**

A particle is moving in a straight line with its acceleration as a function of x given by $\ddot{x} = -e^{-2x}$, where x is its displacement in metres and $t \geq 0$ is the time in seconds.

It is initially at the origin and is travelling with a velocity of 1 metre per second.

(a) Show that its velocity $\dot{x} = e^{-x}$. (3 marks)

(b) Hence show that $x = \ln(t+1)$. (2 marks)

End of questions